BROWN UNIVERSITY LIBRARY NOV 3 1933

math

NOV 2 1933
Club Rates Will be mailed on Application

Mathematics News Letter

SUBSCRIPTION $1.00 PER YEAR
Single Copies..15c

Published eight times per year at Baton Rouge, Louisiana.

Dedicated to mathematics in general and to the following aims in particular: (1) a study of the common problems of secondary and collegiate mathematics teaching, (2) a true valuation of the disciplines of mathematics, (3) the publication of high class expository papers on mathematics, (4) the development of greater public interest in mathematics by the publication of authoritative papers treating its cultural, humanistic and historical phases.

EDITORIAL BOARD

S. T. SANDERS, Editor and Manager, L. S. U., Baton Rouge, La.
H. E. BUCHANAN, Tulane University, New Orleans, La.
T. A. BICKERSTAFF, University, Miss
W. VANN PARKER, Miss. Woman's College, Hattiesburg, Miss.
W. E. BYRNE, Virginia Military Institute, Lexington, Va.
RAYMOND GARVER, University of California at Los Angeles, Los Angeles, Cal.
P. K. SMITH, L. P. I., Ruston, La.
JOSEPH SEIDLIN, Alfred University, Alfred, N. Y.
ALENE THURMAN, Rayville, La.
C. D. SMITH, Miss. State College, State College, Miss.
F. A. RICKEY, Mandeville, La.
IRBY C. NICHOLS, L. S. U., Baton Rouge, La.
H. LYLE SMITH, L. S. U., Baton Rouge, La.
W. PAUL WEBBER, L. S. U., Baton Rouge, La.
WILSON L. MISER, Vanderbilt University, Nashville, Tenn.
MRS. MARTIN L. RILEY, Baton Rouge, La.
JAMES McGIFFERT, Rensselaer Polytechnic Institute, Troy, N. Y.
HERBERT ReBARKER, East Carolina Teachers College, Greenville, N. C.
LILBURNE NICHOLSON DASPIT, L. S. U., Baton Rouge, La.
DOROTHY McCOY, Belhaven College, Jackson, Miss.
DORO M. FORNO, N. O. Normal School, New Orleans, La.

MRS. E. J. LAND, Advertising and Circulation Manager . . . 216 Main Street, Baton Rouge, La.
ALENE THURMAN, Field Agent for Louisiana . . . . . Rayville, La.

VOL. 8 BATON ROUGE, LA., OCTOBER, 1933 No. 1

CONTENTS

Mathematics and the Schools

Mathematics as a Character Builder

Intrinsic Decimals

Alternating Current Calculations

Book Review Department

Problem Department

MATHEMATICS AND THE SCHOOLS

A leader in American mathematics recently mailed out a circular letter, the opening line of which reads: "In these days when mathematics is being attacked on all sides . . . . . ".

Disconcerting as the implied public confession may be to some devoted followers of the queen of the sciences, analysis will show, as it has shown, that promptings to such attacks on mathematics originate largely with a school leadership whose schemes of training for the average youth are heavily loaded with vocational aims. The theories of education offered by this type of leadership are measurably determined by currents of popular opinion or democratic trends rather than by the profounder considerations of culture, science and a discipline mind. A wise speaker was heard to say: "The modern university is a veritable Noah's Ark." How is it possible that mathematics should receive the serious attention that it erstwhile received when along-side of it are so many other materials that have been admitted to school and college curricula avowedly framed to reflect only, not to modify or to mould, current, social and industrial ideals?

The universal spread in school and college of the administrative principle that the student may choose his subjects of study can not be defended as an unmixed good. At best it appears to be little more than a concession to the individualism that is a natural off-spring of democratised institutions. One needs to have done but a small amount of college teaching in order to have observed numerous cases of the student who suffers serious handicap because he made an unwise choice of under-pinning courses.

Few American college and university curricula are found in which the freshman is permitted at his own option to waive further study of his native language. Generally he is required—not permitted to choose—but required, to take at least three hours a week of English for a full year. But, if his option is denied where English is concerned why is it allowed where mathematics or chemistry is concerned? Such discrimination

is evidence that the student's option privilege in subjects of study is not altogether wise or sound—even from the view-point of the administrative mind.

In Louisiana high schools, as in other State secondary systems, the widest student option of subjects prevails. In the seven various four-year-course curricula recently approved and published by the State Department of Education, plane geometry is made optional but algebra is required. If the student's freedom to choose subjects where geometry is concerned is sound, does it cease to be sound where algebra is concerned?

The recent altered status of mathematics in the American school is traceable to no single cause, but to a complex of causes. Those cited are but a few of them. Space does not permit the citation of all. We mention two others. These are: (a) The impression generally entertained by the average utility-minded American citizen that the vast and growing amount of mathematical research that is being turned out can have no more present or potential value to society than has a game of chess or a game of whist or bridge. (b) A widely prevailing inertia among great numbers of mathematics teachers—an inertia plainly indexed in many forms (see Professor W. D. Reeve's circular letter). While rooted in many causes, undoubtedly one of the root causes of this inertia is the fact that far too many mathematics teachers are job-minded or degree-minded rather than mathematics-minded.—S. T. S.

*MATHEMATICS AS A CHARACTER BUILDER

By I. C. NEW
Holmes Junior College, Goodman, Miss.

I am convinced that there is not a study in the modern curriculum that has done more to shape the character of our civilization than mathematics. I am also convinced that no other subject holds a greater potential force for character building. But surveying this force and giving the world a clear view of the character building possibilities of mathematics is not an easy task. The subject is immense; the material is superabundant. It is clear that the task is far beyond the resources of a ten minute discourse, and so it is necessary to restrict and to select.

It is tragical that the average student of today is not led to grasp a fuller meaning of the science. I believe that the teachers of mathematics have failed in giving to the world the proper view and understanding of the science. This, I believe is largely due to the failure of mathematics teachers to comprehend the subject themselves. You well know that there has grown up an attack by a few educational theorists that mathematics is unnecessary in liberal education. I believe that this is a sure indication of mathematics teachers having failed to de liver the message that the science offers. Then, I should say that mathematics teachers in general need a deeper knowledge of the philosophy and the reach of the science. Further, I believe that mathematics as a character builder is a very timely subject and one that should lead us to begin the study of the philosophy of mathematics. If this results, then the random statements in this paper will not have been in vain.

Let us suppose that everything that has any relation to mathematics in this world could be wiped out of existence at this instant. Picture the results. Every mill in the world would be shut down. All business concerns would be forced to close their doors. Every ship on the seas would be stricken with blindness, and would force the starvation of its human burden. No more skyscrapers could be erected. Wall street would close its portals. The engineering world would awaken tomorrow to a living death. Mines would be shut down. Trade would relapse to the days of savagery. Thus, we view the necessity of the public beingg iven information about such an extensively

*Read before Mathematics Section of the M. E. A., April 28, 1933.

used science. This knowledge is necessary to a well-balanced character in the complex life of today. Socrates has said that "badness is a species of ignorance; the wise man, the wise woman, will be good."

I should like to call your attention to the study of the opinions of all who have written regarding the subject of the value of mathematics in liberal education. This is the work of Dr. Florian Cajori of the University of California. No writers for or against were omitted.

		Greek Period	Roman Period Middle Ages	15th and 16th Centuries	17th and 18th Centuries	19th Century	20th Century	TOTAL
Philosophers Theologians Psychologists	For	3	9	6	23	14	11	66
	Against	1	0	0	16	3	6	26
Educators	For		4	2	6	31	19	62
	Against		0	0	1	6	14	21
Mathematicians	For	4	2	10	59	49	66	190
	Against	0	0	0	1	1	3	5
Scientists not Mathematicians	For	3	1	2	4	5	3	18
	Against	0	0	0	1	1	3	4
Literary Men and Statesmen	For	2	4	3	11	24	4	48
	Against	0	0	4	11	8	1	24
Business Men Lawyers, etc.	For						219	219
	Against						48	48
TOTAL	For	12	20	23	103	123	322	603
	Against	1	0	4	30	20	73	128

With a little research, I am convinced that it could be satisfactorily shown, that:

1. Mathematics develops skill in systematizing life's experiences.
2. Mathematics develops stability.
3. Mathematics develops a love for beauty.
4. Mathematics develops love for the truth.
5. Mathematics will lead to a conviction of one's duty to God.

Through long eons of human endeavor, men have been moving forward step by step in the world of ideas, discovering now one, now another of the regularities which nature exhibits. In the effort to comprehend these discoveries and to arrange them into a system of experience men have grown to the point where their minds have evolved

certain general ideas and certain general modes of thought. It could not be perfected until a system of symbols was invented which could express adequately the ideas of precision and regularity.

When we take a look at the great number of graduates of our schools who are lost in society, we question the value of their training. By lost, I mean in a state of confusion. They are unable to systematize life's experiences. Mathematics develops skill in systematizing these experiences. An individual develops intellectually in proportion to his ability to systematize his experiences in life. The development of this ability is the greatest thing that the school can do for the student. The student's confusion is the school's responsibility, and has developed because we do not require rigorous thinking any more.

He who is master of mathematics has a way of thinking that the race has worked out with infinite labor. We think of this individual as being stable in all of his life's relations. We think of this stability as having been developed through the study of our rigorous science. The mathematician never again falls back into the confused and inexact ways of viewing the world. He has grown intellectually by carrying himself to the summit of human vision. He has become a new individual, and is living in an entirely new world. He is living in a world whose atmosphere is saturated with the divine. His personality is aglow with divine understanding.

If a student will apply himself to the study of mathematics, he will develop his power of concentration and patience. He will acquire the habit of meeting life's reverses calmly. You well know that we see this trait in the student who is mathematically trained more than in any other.

Mathematics develops a love for beauty. The beauty of symmetry—where else is it found so completely as in mathematics? Where do you find more rhythm? Poetry or music does not contain more rhythm than mathematics. In fact, mathematics is the base of poetry and music. Nature arranges the leaves of plants in accordance with certain mathematical series (Fivonacci series). The snow crystal recognizes the poetry of the complex sixth roots of unity. Is not the table of logarithms a series of sonnets? Mathematics is the only science that can carry one to the summit of intellectual vision. To take the road of mathematics and journey through the heights of human thought, will give one a love for everything that is beautiful in this life of concepts.

Mathematics (says Claudius Ptolemy) directs the moral conduct of life in the course of everyday events. It elevates and consolidates character. This science tends to sharpen the apprehension and vision.

For in accordance with its recognition of uniformity, strict order, symmetry and simplicity, it brings to its devotees a love for what is most beautiful, and by habit, leads the soul to tune in with all that is good and divine.

If mathematics is properly taught, it becomes an interpreter of life and the world. Mathematics when properly understood will give the mind of the student the conception of an ordered, lawful universe in which all phenomena yield to quantitative investigation, a universe in which the reign of law is absolute, and in which all phenomena in this space-time world fall into order in a cause and effect. When you bring a student to the right conception of the universe, he will seek the ideal life.

One of the greatest responsibilities of the school is to arouse in the student's soul a contemplation of the truths that endure. No science can give one a concept of every known truth more completely than mathematics. One cannot master mathematics without developing a divine respect for the truth. Mathematics gives the student a vision of the ideal. It is only in mathematics that we find ideal justice and satisfaction for our craving for rectitude of thought. It is only through mathematics that we find dominion over the energies and ways of the material universe. It is only in mathematics that we find imperishable beauty, and dignity of peace of intellectual harmony. It is clear enough that mathematics is the most precious response that the human spirit has made to the call of the infinite and eternal. It is man's best revelation of the "Deep Base of the World."

Mathematics helps the student to interpret his economic environment when he leaves school and enters life. But it is of greatest value in building a reliable philosophy. Mathematicians are led to the highest and most exalted pleasures known to mankind. Mathematics makes one become at home in the world. The universe with its changes does not bring the terror to the mathematician that comes to the ignorant. The mathematician becomes intimately familiar with cosmic, economic, and social forces. This familiarity will lead the individual to formulate with precision and to predict with certainty. It is true that teachers satisfy themselves by teaching a system of symbols that have no meaning to the student. This is the great tragedy

of mathematics teaching. Teachers become so absorbed in teaching symbols and manipulations, that they fail to point out the meaning of these symbols and their use in developing rules of conduct.

Who is so unfortunate as not to know something of the religious awe, the solace and peace that comes from contemplation of the purity and everlastingness of mathematical truth? Could we not formulate the whole of cosmic history from eternity to eternity in the minutest detail by a system of differential equations? If so, then the beauty of the universe would shine with dazzling brilliancy in the human mind, and men would become enveloped in a purer and holier vision of life.

Except through the modern mathematical doctrine of infinity, there is no rational way by which we can even approximate an understanding of the supernal attributes with which our faculty of idealization has clothed Deity.

Mathematics is precisely the ideal handling of the problems of of life, and the central ideas of the science, the great concepts about which its stately doctrines have been built up, are precisely the chief ideas with which life must always deal. The mathematical concept of constant and variable are represented familiarly in life by the notions of fixedness and change. The concept of equation or that of equational system, imposing restriction upon variability, is matched in life by the concept of natural and spiritual law, giving us certain freedom from confusion. What is known in mathematics under the name of limit is everywhere present in life in the guise of some ideal. The supreme concept of functionality finds its correlate in the all-pervasive sense of interdependence and mutal determination among the elements of the world. The mathematical concept of invariance and that of infinitude is nothing more than the mathematizations of life's hopes and dreams, of that which has ever been the object of its deepest passion and of its dominant enterprise, I mean the finding of permanance in the midst of change. We might multiply these examples, a correlation that is so abounding and complete as indeed to suggest a doubt which is the juster, to view mathematics as the abstract idealization of life, or to regard life as the concrete realization of mathematics.

Then mathematics develops ideas of abstract beauty, goodness and magnitude. Mathematics will bring the student to see all that is greatest in life and lead him to a desired conception of the immeasurable future. This would bring the individual to our conception of perfec-

tion in character. Sin itself is confusion of the human mind due to the absence of mathematical regulation.

I can think of no branch of human effort that will carry one to a clearer vision of the force that we call *right* in this world than that of our chosen science. It carries one to the depth of ideas, gives him an inner vision of all that is pure and holy. It carries him to the outer realm of infinity giving him a solid spiritual basis, allowing him to behold the fading beauties of time and sense, and holding out to his view the shining celestial City in the white radiance of eternity.

Bibliography

1. Cajori, Florian; "Mathematics in Liberal Education". Christopher Publishing Co.
2. Keyser, C. J.; "Human Worth of Rigorous Thinking". Columbia University Press.
3. "The Third Yearbook, National Council of Teachers of Mathematics". Columbia University Press.
4. "What the Colleges Are Doing". Ginn and Co.
5. Ford, W. B.; Am. Math. Monthly; Vol. 1919.

INTRINSIC DECIMALS

By H. T. R. AUDÉ
Colgate University

In a recent number of the Mathematics News Letter (vol. 7, No. 3) Professor J. McGiffert has an article on intrinsic decimals. In the first part he exhibits the repeating decimal

$$\frac{1}{7} = .142857142857 \ldots\ldots$$

also the decimal equivalent of the other five fractions $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$,

and calls attention to the fact that each of these decimals contains the same six figures in exactly the same order, but with a different beginning. On account of this property the number 7 is called a perfect

repeater. It is then pointed out that perfect decimal repeaters occur only among those rational proper fractions which have a prime number p for denominator, also that there must be $p-1$ figures in the repetend in order that the $p-1$ proper fractions of denominator p may have distinct repetends from the same sequence of figures. Professor McGiffert finally raises the question, why do certain primes, 7, 17, 19, and others, furnish perfect repeaters; while other primes, 3, 11, 13, and others, do not. This question is answered in this paper by giving the necessary and sufficient conditions that p, a prime greater than 5, is a perfect repeater.

In the first place, it is seen that there must be at least $p-1$ figures in the cycle which makes the repetends so that each of the $p-1$ fractions:

$$\frac{1}{p}, \frac{2}{p}, \ldots\ldots\ldots, \frac{p-1}{p}$$

can have a distinct representation. In the case of the perfect repeater 7 if we change the repeating decimal

$$0.142857142857\ldots\ldots\ldots,$$

to a common fraction by the formula for the sum of a geometric progression. There results

$$\overline{0.142857}\ldots\ldots = \frac{0.142857}{1-10^{-6}} = \frac{142857}{10^6-1} = \frac{1}{7}$$

It is noted that 7 divides into 10^6-1 and gives the sequence of six figures 142857 which is the cycle for the six fractions, with 7 for denominator. The prime 11 which is not a perfect repeater is a factor of 10^2-1. The division yields only two figures for the repetend, and it is impossible to express cyclicly the ten different fractions with 11 for denominator. To aid precise statements, let k dencte the smallest integer for which the expression 10^k-1 is divisible by p; then repeating the previous two statements we have 7 is a factor of 10^k-1 for $k=6$ and 11 is a factor of 10^k-1 for $k=2$. Continuing with this notation it is easily seen that 13, which is not a perfect repeater, is a factor of 10^k-1 for $k=6$, while 17, a perfect repeater, is a factor of 10^k-1 for $k=16$. From Fermat's general theorem it is known that every prime number p ($p>5$) is a factor of $10^{p-1}-1$. It can therefore

be stated that *a necessary condition to obtain the* $p-1$ *figures for the repetend so that the prime number* p *can be a perfect repeater is that* p *is a factor of* 10^k-1 *for* $k=p-1$ *and for no smaller value of* k.

This will also be seen to be sufficient. For if p divides into 10^i-1 for $i=p-1$, but for no smaller value of i, then write

$$\frac{10^{p-1}-1}{p} = q, \text{ an integer} \qquad (1)$$

By adding zeros on the left of the figures of the quotient q until there is exactly $p-1$ figures there results a sequence of $p-1$ figures which denote by the symbol $\bar{q}$. Solving (1) for $\frac{1}{p}$ gives the equation

$$\frac{1}{p} = \frac{q}{10^{p-1}-1} = 0.\bar{q}\,\bar{q} \ldots\ldots \qquad (2)$$

Since 10^i-1 is divisible by p for $i=p-1$, but is not divisible by p for $i=1,2,3,\ldots\ldots,p-2$, therefore dividing 10^i by p yields a quotient q_i and a remainder r_i as indicated.

$$\frac{10^i}{p} = q_i + \frac{r_i}{p} \text{ for } i=1,2,\ldots\ldots,p-1. \qquad (3)$$

Equation (3) includes the case when the exponent $i=p-1$ and the remainder is 1, which follows directly from Fermat's theorem and is given in equation (1).

Continuing with the proof, it will be seen *that none of the remainders* r_i *in* (3) *are equal.* For if $r_j=r_i$ and $i<j<p$, then the following relation should hold

$$\frac{10^j-10^i}{p} = q_j-q_i, \text{ an integer.}$$

Or

$$\frac{10^i(10^{j-i}-1)}{p} = \text{an integer.}$$

But this is impossible for p cannot be a factor of $10^{j-i}-1$, since $j-i$ is less than p.

It follows that *the* $p-1$ *different remainders* r^i $(i=1,2,\ldots\ldots p-1)$ *in equation* (3) *are equal in some order to the integers* $1,2,\ldots\ldots,p-1$. Therefore each one of the fractions

$$\frac{1}{p},\ \frac{2}{p},\ \ldots\ldots\ldots,\ \frac{p-1}{p} \tag{4}$$

can be obtained from (3) by using a proper corresponding value of i. Equation (3) thus solved becomes

$$\frac{r_i}{p} = \frac{1}{p}\ .\ 10^i - q_i \tag{4}$$

It remains to show that all of the fractions in (4) when written as decimals have their repetends from the same sequence of figures in cyclic order. They will be distinct however by each beginning the cycle in a different place. That this is true will be seen by first taking the decimal value of $\frac{1}{p}$ given in equation (2) and substituting it in equation (5) giving

$$\frac{r_i}{p} = 10^i\ .\ 0\ .\ \bar{q}\,\bar{q}\ldots\ .\qquad -q_i \tag{6}$$

The left member $\frac{r_i}{p}$ is a proper fraction, and so is the result on the right side. After performing the operations: first, the multiplication by 10^i by moving the decimal point i places to the right, followed by the subtraction of q_i which must cancel the integral part, there results a proper decimal repeating fraction with the $p-1$ figures of its repetend taken from the sequence named $\bar{q}$ and in that same order, but the cycle begins with the figure which is in the $i+1$st place. The repeating part of each and every fraction $\frac{r_i}{p}$ can thus be obtained from the se-

quence which is the repetend for the fraction $\frac{1}{p}$ by counting i places to the right and beginning the cycle of the $p-1$ figure sequence at that place. *Therefore if* p, *a prime greater than* 5, *is a factor of* 10^i-1 *for* $i \not< p-1$, *then and only then is* p *a perfect decimal repeater.* Which in the language of the theory of numbers is the same thing as stating, *if* 10 *is a primitive root of* p $(p>10)$, then and *only then is* p *a perfect repeater.* For $p=7$ then $10-p=3$ is a primitive root of p.

ALTERNATING CURRENT CALCULATIONS

By W. E. BYRNE

In the introductory courses in alternating current theory more attention is now being given to the differential equations associated with simple circuits. However, the student is referred to differential equation texts which treat the engineering applications only in passing. Often the transition from the classical exposition of the mathematician to the vector representation and discussion of the engineer is so vague that the beginner loses trace of the essential reasoning. It is the aim of this article to bring out the necessary links in the transition.

The following results of complex number theory and of operational calculus are assumed known:

Any complex number may be written in the form

$$z=x+iy \tag{1}$$

(x, y real and $i^2=-1$). If the point M having the rectangular coordinates (x,y) is joined to the origin the vector OM may be used to picture z. If (ρ,θ) are the polar coordinates of M $(e \not< 0)$

$$\rho=\sqrt{x^2+y^2}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}, \quad \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

The knowledge of cos θ and sin θ does not determine θ closer than a multiple of 2π radians unless other restrictions are added. z may be rewritten as

$$z = \rho(\cos\theta + i\sin\theta) = \rho e^{i\theta} \tag{2}$$

If z_1 and z_2 are two complex numbers

$$z_1 = x_1 + iy_1 = \rho_1 e^{i\theta_1}$$

$$z_2 = x_2 + iy_2 = \rho_2 e^{i\theta_2}$$

then the product $z_1 z_2$ is given by

$$z_1 z_2 = \rho_1 \rho_2 e^{i(\theta_1 + \theta_2)} \tag{3}$$

and the quotient by

$$z_1/z_2 = (\rho_1/\rho_2)e^{i(\theta_1 - \theta_2)} \text{ if } \rho_2 \neq 0. \tag{3'}$$

The *real part* of z, indicated by Rz, is x. Also

$$Rz = R\rho e^{i\theta} = \rho\cos\theta = \rho\sin\left(\theta + \frac{\pi}{2}\right) \tag{4}$$

The *modulus* of z is the length of the associated vector OM. It is indicated by the symbol $|z|$.

$$|z| = \rho. \tag{5}$$

If D indicates the operation d/dt and if f(D) is a polynomial in D with constant real coefficients, a particular integral of the differential equation

$$f(D)u = F(t) \tag{6}$$

is indicated by

$$(1/f(D))\ F(t) \tag{7}$$

In particular

$$(1/f(D))\sin\omega t = R\ \frac{1}{i}\ \frac{e^{i\omega t}}{f(i\omega)} \tag{8}$$

and

$$(1/f(D))\cos\omega t = R\ \frac{e^{i\omega t}}{f(i\omega)} \tag{9}$$

if ω is a constant and if $f(i\omega) \neq 0$.

Problem 1. Circuit with resistance R and inductance L in series where the applied electromotive force is of the form

$$A \sin \omega t + B \cos \omega t$$

(R, L, A, B, ω constants; R, L, ω positive)

The differential equation involved is

$$L \frac{dI}{dt} + RI = A \sin \omega t + B \cos \omega t \tag{10}$$

where I is the instantaneous value of the current.

The second member of (10) may be rewritten

$$\sqrt{A^2+B^2} \sin (\omega t+\alpha) = E_m \sin (\omega t+\alpha) \tag{11}$$

with

$$\cos \alpha = \frac{A}{\sqrt{A^2+B^2}}, \quad \sin \alpha = \frac{B}{\sqrt{A^2+B^2}}$$

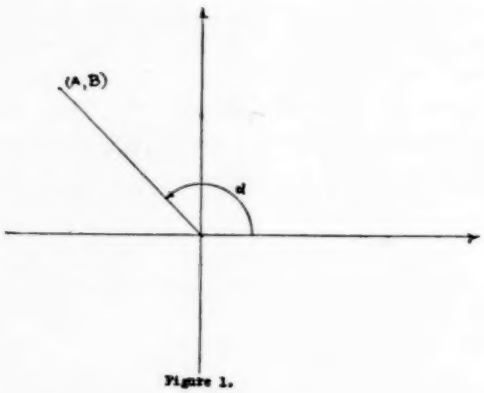

Figure 1.

Here the differential operator is given by

$$f(D) = LD + R$$

and the general solution of (10) is

$$I = Ke^{-\frac{Rt}{L}} + \frac{1}{f(D)} E_m \sin (\omega t+\alpha) \tag{12}$$

with K a constant of integration.

If $f(i\omega) = Li\omega + R$ is written in the form

$$f(i\omega) = z = re^{i\gamma} \tag{13}$$

a short calculation shows

$$\frac{1}{f(D)} E_m \sin(\omega t + \alpha) = R \frac{E_m e^{i(\omega t + \alpha}}{i\, f(i\omega)} = \frac{E_m}{r} \sin(\omega t \alpha - \gamma) \tag{14}$$

since

$$\frac{E_m e^{i(\omega t + \alpha)}}{i\, f(i\omega)} = \frac{E_m e^{i(\omega t + \alpha)}}{i\gamma e^{i\gamma}}$$

has for its modulus $\frac{E_m}{r}$ and for its angle $\omega t + \alpha - \gamma - \frac{\pi}{2}$ (apply formula 4).

Hence

$$I = Ke^{-\frac{Rt}{L}} + \frac{E_m}{|z|} \sin(\omega t + \alpha - \gamma) \tag{15}$$

The quantity z which enters into the calculations is called the ***impedance*** of the circuit for the frequency $\frac{\omega}{2\pi}$. The first term in (15) disappears for all practical purposes after a few cycles, and is called the *transient* term; the second term represents the ***steady state.***

Electrical engineers picture the relative magnitudes and phase relations as well as the instantaneous values of the impressed sinusoidal voltage and the resulting steady state current by associating with the voltage the complex number

$$E^* = E_m e^{i(\omega t + \alpha)} \tag{16}$$

and with the current the number

$$I^* = \frac{E^*}{z} \tag{17}$$

so that their projections on the axis of imaginaries give the instantaneous values. (In practice the length of these vectors is taken as a

certain factor times the maximum voltage or current so that root mean square values may be read from the diagram, but this does not alter the relative size or phase relations and is equivalent to a change in scale.)

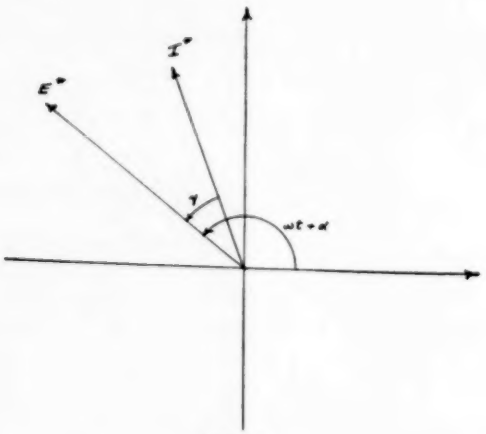

Figure 2.

If only relative values, not instantaneous values, are needed, t may be given a convenient value, say 0.

Problem 2. Circuit with resistance R and capacitance C in series, applied e. m. f. $E_m \sin(\omega t + \alpha)$

(R, C, E_m, ω positive constants).

Here the differential equation is

$$RI + \frac{1}{c}\left| q_0 + \int_0^t I\,dt \right| = E_m \sin(\omega t + \alpha) \tag{18}$$

and q_0 is the charge at time $t = 0$ on the condenser.

Differentiating (18) yields

$$R\frac{dI}{dt} + \frac{1}{c} I = E_m\, \omega \cos(\omega t + \alpha) \tag{19}$$

Here $f(D) = RD + \frac{1}{c}$

The solution of (19) is

$$(20) \qquad I = Ke^{-\frac{t}{RC}} + \frac{1}{f(D)} E_m \omega \cos(\omega t + \alpha)$$

By writing $f(i\omega) = Ri\omega + \frac{1}{c} = i\omega\left(R + \frac{1}{i\omega c}\right) = i\omega z = i\omega r e^{i\gamma}$

a calculation similar to that previously made shows that

$$(21) \qquad I = Ke^{-\frac{t}{RC}} + \frac{E_m}{|z|} \sin(\omega t + \alpha - \gamma)$$

The vector representation is

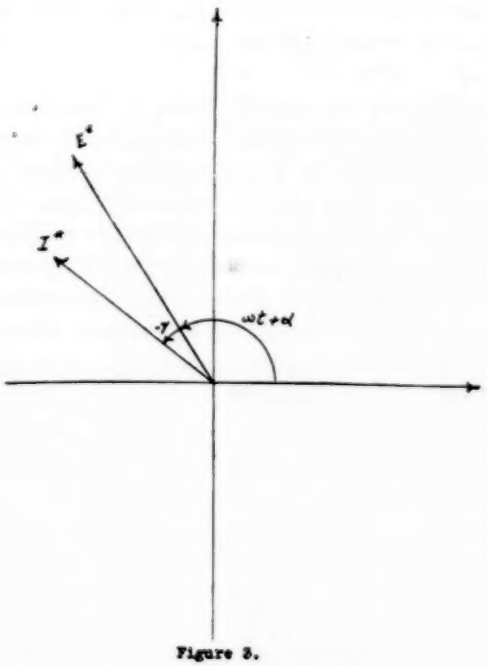

Figure 3.

(Note that γ may be taken as an angle in quadrant IV).

Conclusion: The preceding examples show that the use of complex numbers in connection with "D" operators has the advantage of

introducing directly the concept of impedance, thus avoiding tedious juggling in the discussion of circuits involving resistance, inductance and capacitance in series or parallel combinations. The impedance enters as a factor of $f(i\omega)$. The simple passage from the formal solutions in sinusoidal form to the vector representation is another advantage of the method.

BOOK REVIEW DEPARTMENT

Edited by
P. K. SMITH

Plane Trigonometry. By W. L. Hart. D. C. Heath and Company, 1933.

An important feature of this book is its flexibility. It may be used without loss of continuity for a short course by omitting certain topics.

The trigonometry of the acute angle and its applications are given before the general definitions of the functions. The writer has found that freshmen have difficulty in taking the general definitions first. The author points out that the introductory part of the chapter on the general definitions may be taken up at first if desired.

Chapter II, twenty pages in length, is devoted to the theory of logarithms. The discussions of the rules of interpretation are effective and clear. Summaries of formulas and method are given to aid the student in organizing his acquired material. Also, review exercises appear at vantage points throughout the text. Only answers to the odd-numbered problems are given. However, answers to the even-numbered problems will be furnished with the text to the student if the instructor so desires it. Answers are given with use of both four and five place tables. The author uses a boxing arrangement in the solutions of triangles which is an aid in the attainment of speed and accuracy. Finally, in this paragraph of miscellaneous features, it should be mentioned that Chapter XII is devoted partly to analytic trigonometry for the benefit of those students who might study higher mathematics.

This appears to be a splendid text book. The mechanical makeup is well done. Bold type is used in places for emphasis. The color is dark red. There are 176 pages and the size is 6″x9″.

Solid Geometry. By A. M. Welchons and W. R. Krickenberger. Ginn and Company, 1933.

In the foreword the authors point out that the text conforms to the recommendations of both the College Entrance Examination Board and of the National Committee on Mathematical Requirements. By a convenient lettering of theorems and corollaries those propositions required by each of the above bodies are indicated. By properly pursuing this lettering throughout the texts may be used to offer a minimum, a medium, or a maximum course. The authors have attempted to meet the problem of varying ability among individual students as well as classes by offering the above grades of courses. At the ends of the chapters two groups of exercises are given. The first group is given for all members of a class; the second for those of superior ability.

There are other interesting features of this text. At the end of the various chapters summaries of the methods of proof and the formulas developed are given. Also at the ends of the chapters three tests are given, viz., a true and false test, a completion test and an application test. In the introduction a set of definitions and postulates and a summary of the methods of proof from Plane Geometry are given. At the end of the text a general summary of the methods of proof of solid Geometry, a table of square roots and cube roots, a brief discussion of logarithms, a table of logarithms, a table of natural trignometric functions, and a general table of solid Geometry formulas are given.

The authors give the old definition: A plane is a surface such that a straight line joining any two points of the surface will lie wholly in the surface. No definition has ever told one just what a plane is. Why try to give a definition? With a sufficient number of illustrations the students will know what a plane is. Again the text has so much in it that it will be difficult to manage. It will take a teacher of study purposes to carry out the course. Will the students do all of the self-testing at the chapter ends? The writer thinks the text is better suited to high school than to college use.

The mechanical makeup and the figures are excellent. The color is orange, the length is 251 pages, and the size is 5†″x8″.

PROBLEM DEPARTMENT

Edited by
T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

Solutions

No. 34. Proposed by R. W. Mattoon, Antioch College, Yellow Springs, Ohio:

Given that two of the angle bisectors of a triangle are equal, to prove by Plane Geometry that the triangle is isosceles.

Suggestion: By C. S. Whitney, Miami, Okla.

This problem is proved in "College Geometry" by Altschiller-Court on page 66 by the Reductio ad Absurdum method.

Problems for Solution

No. 35 corrected: Proposed by Earl Thomas, L. S. U.:

The ends of two ladders rest at the bottoms of parallel vertical walls on opposite sides of an alley and each leans against the wall on the opposite side of the alley. The first is 40 feet long, the second is 30 feet long and they cross at a point 10 feet above the alley. How wide is the alley?

***

No. 41. Proposed by T. A. Bickerstaff:

In problem 35 find the path of the crossing point of the ladders as the width of the alley varies from 0 to 30 feet.

***

No. 42. Proposed by T. A. Bickerstaff:

Twelve cards have been dealt, six down, and the other six include a jack, two kings, a seven, a five, and a four. What is the probability that the next card will be a four or less?

No. 36. Proposed by William Sell, University, Ala.:

Given points A and B on the circumference, and the line l tangent, construct the circle using ruler and compasses.

Solved by JAMES A. WARD, L. S. U.:

The solution of Problem 36 given in the Teacher's Edition of Wentworth's "Plane and Solid Geometry" published in 1900 as follows:

"Given: Points A and B and a line L

"To construct: A circle passing through A and B tangent to L.

"Construction: Draw AB cutting L at C. Find the mean proportional between AC and BC. Lay off CD equal to this mean proportional. Draw OD perpendicular to L. Draw 00′ bisecting AB at right angles. At 0, the intersection of 00′ and OD, describe a circle with radius OA. This circle will pass through A and B and be tangent to L.

"Proof: Since 0 is equally distant from A and B, the circle will pass through B. By construction, CD is the mean proportional between AC and BC. Therefore, the circle is tangent to L, by the theorem: If from a point without a circle a secant and a tangent be drawn, the tangent is the mean proportional between the secant and its external segment.

Q. E. F.

"Discussion: By taking CE equal to CD a second solution may be found."

Now there are several things in this problem which Wentworth overlooked. Suppose AB did not meet L. What would you do then? Or suppose A and B were on opposite sides of L. You would follow all his instructions but you would certainly not have a circle tangent to L. Let us prove the theorem again and treat these and other similar difficulties.

Given: Points A and B and the line L, all in the same plane.

To construct: A circle through A and B tangent to L.

A and B will be, I, coincident or II, distinct. Case I, if A and B are coincident, they will be, (1) on L or (2) not on L.

(1) If they are on L, draw a line through A perpendicular to L. Then with any point 0 on this line as a center and OA as a radius draw a circle. It will be tangent to L and pass through A. There is an infinite number of solutions.

(2) If A and B are coincident and not on L, draw any line through A intersecting L at C. Draw line m, the perpendicular bisector of the segment AC. Draw a line through C perpendicular to L. This will intersect m at some point 0, because AC is not parallel to L. With 0 as center and OA as radius describe a circle. This will be tangent to L because it is equidistant from A and C and on a line through C perpendicular to L. There is an infinite number of solutions, one for every C.

Case II, in which A and B are distinct. The line AB may (1) coincide with L, (2) be parallel to L, or (3) intersect L.

(1) The case in which AB coincides with L. Any circle through A and B will necessarily intersect L at A and B. A tangent line can intersect a circle in only one point. Hence there can be no solution.

(2) AB parallel to L. Construct the perpendicular bisector of the segment AB. It will intersect L at some point D. Draw AD. Draw the perpendicular bisector of the segment AD. The segments AD and AB are non-parallel chords. Their perpendicular bisectors will meet at O, the center of the circle through A, B, and D. The perpendicular bisector of AB will be perpendicular to L. Therefore the circle with center O will be tangent to L. This is the required circle.

(3) In which AB intersects L at some point C. The notation can be so chosen that (a), C will coincide with B; (b), C will be between A and B, or (c), B will be between A and C.

(a) In which C coincides with B. This differs from Case I—(2), above, only in that C is fixed by the given point B here, while in the other, C is the intersection of L and any line through A. With C chosen the solution is the same. In this case there is only one solution.

(b) The case in which C is between A and B. The segment AB will be a chord of any circle drawn through A and B. C is any point of this segment between A and B. Hence any line through C will

intersect the circle in two points. L is such a line. Hence, L cannot be a tangent. Hence there is no solution.

(c) The case in which B is between A and C. This is the case solved by Wentworth as given above.

We have discussed the following cases:

I—A and B coincident

(1) A and B on L ..... infinite number of solutions

(2) A and B not on L ..... infinite number of solutions

II—A and B distinct

(1) AB coincident with L ..... no solution

(2) AB parallel to L ..... one solution

(3) AB intersects L at C .....

(a) C coincident with B ..... one solution

(b) C between A and B ..... no solution

(c) B between A and C ..... two solutions

Out of these 7 possible cases Wentworth gave only one and gave it in such a way that it seemed to be the only case with the given hypothesis. Many high school students will be able to readily see some of these given above, and with a little encouragement could pick them all. It seems a pity that problems of this kind pass entirely unnoticed.

Note: I have used the "between" idea in giving the positions of A, B, and C in the latter part. It seems to be much easier to understand than the customary "dividing internally" and "dividing externally".

***

Also solved by Henry Schroeder, Ruston, La., Robert C. Yates, University of Maryland, C. S. Whitney, Miami, Okla., and the proposer.

No. 37 Proposed by William Sell, University, Ala.:

Given point A on the circumference, line l tangent, and line c containing the center. Construct the circle using ruler and compasses.

Solved by JAMES A. WARD, L. S. U.:

In this problem there are several cases. A will be (1) on C or (2) not on C.

(1) If A is on C it will be (a) at the intersection of C and L, or (b) not at the intersection.

(a) If A is at the intersection of c and L, draw a line through A perpendicular to L. If this line coincides with C, any point O on it as a center and OA as a radius will fulfill the conditions. There is an infinite number of solutions. If the perpendicular does not coincide with C, there will be no solution.

(b) If A is on C and not at the intersection of C and L. Draw a line m through A perpendicular to C. It will (x) be parallel to L or (y) intersect L.

(x) If the line is parallel to L, C is perpendicular to L and will intersect L at some point D. Bisect the segment AD at O. With O as center and OA as radius describe a circle. It will be tangent to L at D and be a single solution.

(y) If m intersects L at some point D, use DA as radius and describe a circle around D which will intersect L at two points, E and F. At E draw perpendicular to L which will intersect C at G. It will intersect C because C is not perpendicular to L. With G as center and GA as radius describe circle. This will go through E and A respectively because GA and GE are equal since they are tangents to the same circle center D from an external point. There is another solution, for the circle center D will intersect L at F also and another circle center H may be found in the same manner. Thus we have two solutions.

(2) If A is not on C, draw a line through A perpendicular to C intersecting C at Q. Lay off 2D equal to QA. Any circle with its center on C passing through A will also pass through B. Therefore our problem reduces to No. 36 Case II, above.

Also solved by Henry Schroeder, C. S. Whitney, and the proposer.

He Wanted Mathematics

He was taking his major work in English, but he wanted mathematics also. Why? This was his answer. He wished "the kind of training that mathematics gave in order to do two things, one, to prune, and the other, to clarify, his English style".

Results? It was later said of him, "One of the most brilliant teachers of English in the country. He has, also, a style which is an inspiration to his pupils, a daily example of what is best in oral and written expression."

LOUISIANA STATE UNIVERSITY

Baton Rouge